Heat Transfer in a Round Tube with Sinusoidal Wall Heat Flux Distribution

CHIA-JUNG HSU

Brookhaven National Laboratory, Upton, New York

Sufficiently accurate values of first twenty eigenvalues, eigenfunctions R_n (1), and the coefficients for series expansion, as well as asymptotic expressions for these quantities, have been obtained for heat (or mass) transfer to fully developed laminar flow inside a round tube with uniform wall heat (or mass) flux. The first ten eigenfunctions are shown graphically for the radius range $0 \leq r/r_0 \leq 1$.

These quantities are used to calculate the Nusselt numbers for sinusoidal wall heat flux distribution and are compared with the corresponding slug flow Nusselt numbers.

In heat removal from nuclear reactors, the heat flux along a coolant channel can, under certain conditions, be approximated by a sinusoidal distribution. The problem of laminar heat transfer to a fluid flowing inside a round tube with wall heat flux varying in a sinusoidal manner has been studied by Dzung (*3*). His approach, although sound, requires the eigenvalues and the associated constants to be evaluated according to approximate asymptotic solutions. It is possible to solve the same problem in a more straightforward manner by utilizing the thermal-entrance-region solution for uniform heat flux which was published by Siegel, Sparrow, and Hallman (*7*). In this approach, however, the first seven, and only published, eigenvalues and the related constants obtained by these authors are quite insufficient, since the infinite series appearing in the solution of the problem converges slowly. To obtain the accurate solution to the problem, values of the higher eigenvalues and the related constants must be available. The same need undoubtedly occurs in dealing with other arbitrary wall heat flux variations, by making use of the solution for the case of uniform wall heat flux.

The first objective of this paper is to report accurate values of the first twenty eigenvalues, eigenfunctions $R_n(1)$, and the coefficients of the series expansion, as well as asymptotic expressions for these quantities, for calculating the entrance-region heat transfer in laminar flow through a round tube with arbitrary wall heat flux. The second objective is to show the variation in the heat transfer coefficient for heat transfer to a fluid in laminar, or slug, flow through a round tube with truncated sinusoidal wall heat flux.

TABLE 1. EIGENVALUES AND THE CONSTANTS

n	β_n^2	$R_n(1)$	C_n
1	25.679611	—0.49251658	0.40348318
	(25.6796)	(—0.492517)	(0.403483)
2	83.861753	0.39550848	—0.17510993
	(83.8618)	(0.395508)	(—0.175111)
3	174.16674	—0.34587367	0.10559168
	(174.167)	(—0.345872)	(0.105594)
4	296.53630	0.31404646	—0.073282370
	(296.536)	(0.314047)	(—0.0732804)
5	450.94720	—0.29125144	0.055036482
	(450.947)	(—0.291252)	(0.0550357)
6	637.38735	0.27380691	—0.043484355
	(637.387)	(0.273808)	(—0.043483)
7	855.849532	—0.25985296	0.035595085
	(855.850)	(—0.259852)	(0.035597)
8	1106.329035	0.24833186	—0.029908452
9	1388.822594	—0.23859024	0.025640098
10	1703.3278521	0.23019903	—0.022333685
11	2049.843045	—0.22286280	0.019706916
12	2428.366825	0.21637034	—0.017576456
13	2838.898142	—0.21056596	0.015818436
14	3281.436173	0.20533190	—0.014346369
15	3755.980271	—0.20057716	0.013098171
16	4262.529926	0.19623013	—0.012028202
17	4801.084748	—0.19223350	0.011102223
18	5371.644444	0.18854081	—0.010294071
19	5974.208812	—0.18511389	0.0095834495
20	6608.777727	0.18192104	—0.0089543767

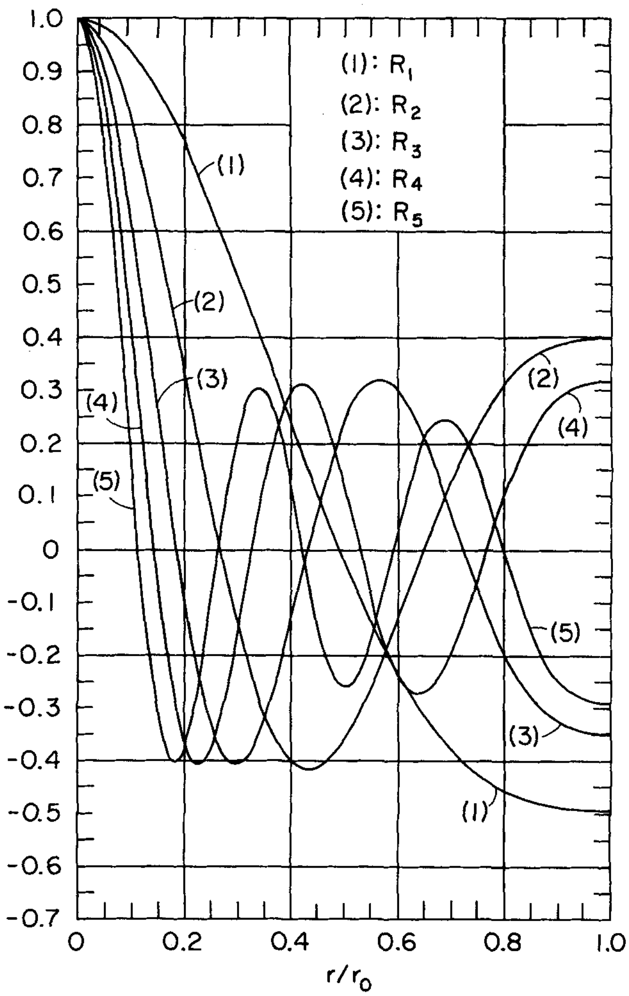

Fig. 1. Calculated eigenfunctions for $n = 1$ to 5.

NUSSELT NUMBERS FOR SINUSOIDAL HEAT FLUX DISTRIBUTION

By assuming a constant property, nondissipative flow, and negligible axial conduction, the solution of the present problem depends on the solution of the equation

$$\rho u C \frac{\partial T}{\partial x} = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] \tag{1}$$

where the following initial and boundary conditions are satisfied

$$\text{at } x = 0 \qquad T = T_o$$

$$\text{for } x > 0 \quad \frac{\partial T}{\partial r} = 0 \text{ at } r = 0 \tag{2}$$

$$k\frac{\partial T}{\partial r} = q_o \sin\frac{\pi x}{L} \text{ at } r = r_o$$

$$(0 < x < L)$$

Laminar Flow

If the flow inside a round tube is laminar, the velocity term u in Equation (1) is replaced by $u_m[1 - (r/r_o)^2]$. The solution to the resulting equation can be obtained in a straightforward manner by applying Duhamel's superposition theorem to the entrance-region solution for a uniform wall heat flux (7). By taking cognizance of the fact that the wall heat flux is sinusoidal, the following expression for local and average Nusselt number can ultimately be derived.

$$Nu_L = \frac{q_o \sin\frac{\pi x}{L}}{(T_w - T_b)}\frac{2r_o}{k} =$$

$$\frac{2\sin\frac{\pi x}{L}}{-\sum_{n=1}^{\infty} C_n R_n(1)\frac{a_n}{1 + a_n^2}\left[\left(a_n \sin\frac{\pi x}{L} - \cos\frac{\pi x}{L}\right) + \exp\left(-\frac{a_n \pi x}{L}\right)\right]} \tag{3}$$

and

$$\overline{Nu} = \frac{q_{total} 2r_o}{(T_w - T_b)_{av} k} =$$

$$\frac{4}{-\sum_{n=1}^{\infty} C_n R_n(1)\frac{a_n}{1 + a_n^2}\left[2a_n + \frac{1}{a_n}(1 - e^{-\pi a_n})\right]} \tag{4}$$

The infinite series appearing in the denominator of both Equations (3) and (4) are slowly converging series. This is mainly because the product $C_n R_n(1)$ decreases slowly as n is increased. Apparently, a large number of this value, together with the eigenvalues, is needed in order to assure the convergence of the infinite series. Evaluation of these quantities will be discussed in the next section.

Slug Flow

In the case of slug flow, the term u in Equation (1) becomes constant, and similar mathematical manipulation leads to the following expression for the local and average Nusselt number.

$$Nu_L = \frac{\frac{\pi}{4}\left(\frac{DPe}{L}\right)\sin\frac{\pi x}{L}}{\sum_{n=1}^{\infty}\frac{1}{1 + b_n^2}\left[\left(b_n \sin\frac{\pi x}{L} - \cos\frac{\pi x}{L}\right) + \exp\left(-\frac{b_n \pi x}{L}\right)\right]} \tag{5}$$

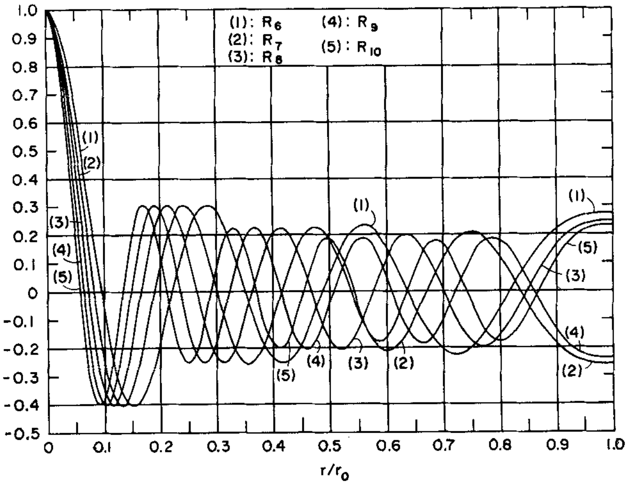

Fig. 2. Calculated eigenfunctions for $n = 6$ to 10.

$$\overline{Nu} = \frac{\frac{\pi}{2}\left(\frac{DPe}{L}\right)}{\sum_{n=1}^{\infty}\frac{1}{1 + b_n^2}\left[2b_n + \frac{1}{b_n}(1 - e^{-\pi b_n})\right]} \tag{6}$$

where $b_n = \frac{4\alpha_n^2}{\pi}\left(\frac{L}{DPe}\right) = \alpha_n^2/N_{Gz}$, and α_n are the positive roots of $J_1(\alpha) = 0$. In this case, again, the infinite series appearing in the denominator of both Equations (5) and (6) converges slowly. Many roots of the Bessel function must be taken in order to obtain an accurate solution.

EVALUATION OF THE EIGENVALUES AND THE CONSTANTS

The main difficulty in obtaining the solution for eigenvalue problems often lies in finding the eigenvalues and the associated quantities. For laminar heat transfer in a round tube with uniform wall temperature, the first eleven eigenvalues and the constants were not determined accurately until a few years ago (1), although Graetz first developed the method of solution in as early as 1883. It is a well-known fact that the precise determination of the higher eigenvalues (large n) according to Graetz's classical approach is extremely laborious.

For the case of laminar heat transfer in a round tube with uniform heat flux, the temperature distribution within the fluid in the thermal entrance region has been obtained, after superimposing the solution for the fully developed region to a conveniently defined temperature variable as

$$\frac{T - T_o}{q_w r_o/k} = \frac{4(x/r_o)}{RePr} + \left(\xi^2 - \frac{\xi^4}{4} - \frac{7}{24}\right) +$$

$$\sum_{n=1}^{\infty} C_n R_n(\xi) \exp\left[-\frac{\beta_n^2}{RePr}\frac{x}{r_o}\right] \quad (7)$$

In the preceding equation, β_n and $R_n(\xi)$ are the eigenvalues and eigenfunctions, respectively, of the following characteristic equation.

$$\frac{d}{d\xi}\left[\xi\frac{dR_n}{d\xi}\right] + \beta_n^2 \xi(1-\xi^2)R_n = 0 \quad (8)$$

satisfying the homogeneous boundary conditions

$$\frac{dR_n}{d\xi} = 0 \quad \text{at } \xi = 0 \text{ and } \xi = 1$$

The coefficients of series expansion C_n in Equation (7) can be calculated through the relationship

$$C_n = \frac{\int_0^1 \xi(1-\xi^2)\left(\frac{7}{24} - \xi^2 + \frac{\xi^4}{4}\right) R_n d\xi}{\int_0^1 \xi(1-\xi^2)R_n^2\, d\xi} \quad (9)$$

which is obtained after utilizing the orthogonal property of the eigenfunctions.

As pointed out earlier, β_n, $R_n(1)$, and C_n had been determined only up to $n = 7$, and they will not suffice to yield a converged solution to both Equations (3) and (4).

In the present study, calculation of these quantities was carried out for $n = 1$ through 20, with the aid of an IBM 7094 computer. The method of Runge-Kutta was used to solve Equation (8) numerically and the eigenvalues determined by trial and error procedure, by using double precision statements. In computing the coefficients C_n the integral appearing in the numerator of Equation (9) was simplified as follows.

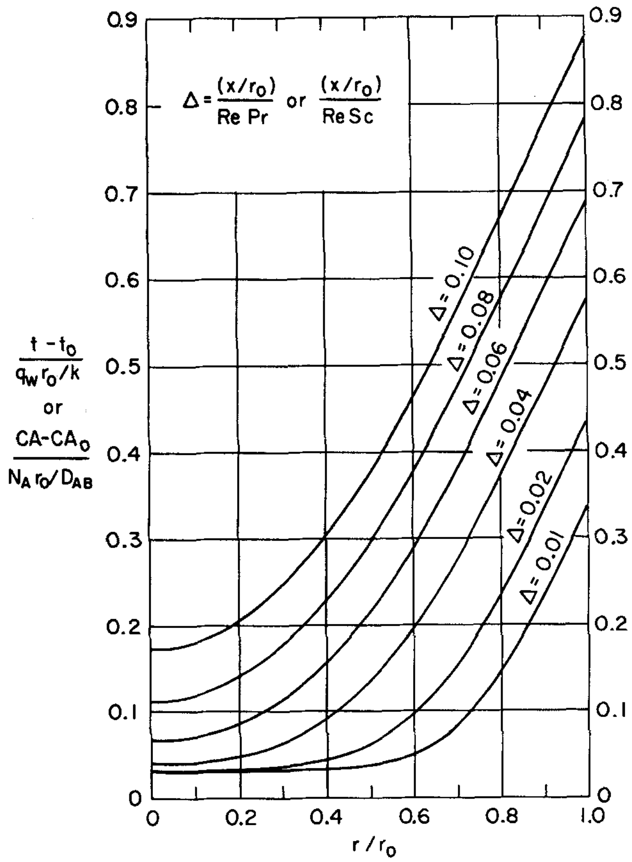

Fig. 3. Entrance region temperature or concentration profiles for laminar flow in a round tube with uniform wall heat or mass flux.

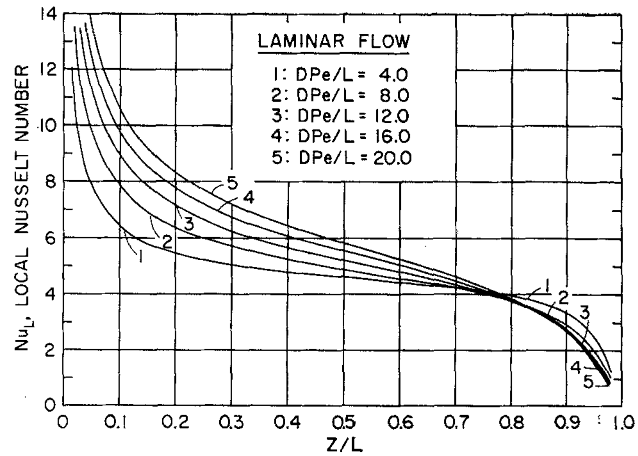

Fig. 4. Local Nusselt number vs. x/L for laminar flow with sinusoidal wall heat flux distribution.

First, combining the integral with Equation (8) and then integrating twice by parts, one obtains

$$\int_0^1 \xi(1-\xi^2)\left(\frac{7}{24} - \xi^2 + \frac{\xi^4}{4}\right) R_n d\xi = -\frac{R_n(1)}{\beta_n^2} \quad (10)$$

Accordingly, Equation (9) can alternatively be written as

$$C_n = \frac{-R_n(1)/\beta_n^2}{\int_0^1 \xi(1-\xi^2)R_n^2 d\xi} \quad (11)$$

The above equation was used to calculate the C_n coefficients in this study. In fact, the integral appearing in the denominator of Equation (11) can also be simplified to the following form by following Graetz's approach (4).

$$\int_0^1 \xi(1-\xi^2)R_n^2 d\xi = \frac{1}{2\beta_n}\left[\frac{\partial R}{\partial \beta}\frac{\partial R}{\partial \xi} - R\frac{\partial^2 R}{\partial \beta \partial \xi}\right]_{\substack{\xi=1\\ \beta=\beta_n}} = -\frac{R_n(1)}{2\beta_n}\left(\frac{\partial^2 R}{\partial\beta\partial\xi}\right)_{\substack{\xi=1\\ \beta=\beta_n}} \quad (12)$$

Thus, another alternative expression for the C_n is

$$C_n = \frac{2}{\beta_n\left(\frac{\partial^2 R}{\partial\beta\partial\xi}\right)_{\substack{\beta=\beta_n\\ \xi=1}}} \quad (13)$$

For the computational procedure adopted here for determining the eigenvalues and eigenfunctions, it is more feasible to use Equation (11) to calculate the C_n coefficients than Equation (13), since precise evaluation of the derivative appearing in the denominator of Equation (13) by numerical method is rather difficult, particularly for the first few eigenvalues.

The twenty calculated values of β_n, $R_n(1)$, and C_n are tabulated in Table 1. The values shown in parentheses are those reported by Siegel, Sparrow, and Hallman (7). It can be seen that the two are in very good agreement. In Figures 1 and 2, the calculated first ten eigenfunctions are shown graphically from $\xi = 0$ to 1. By using these

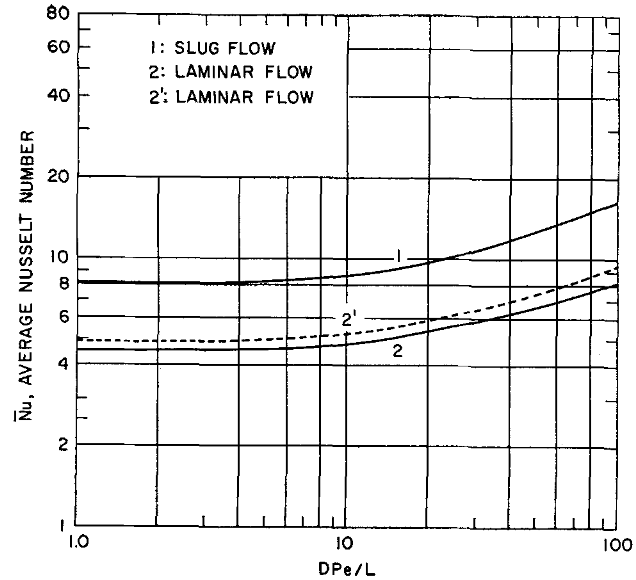

Fig. 5. Average Nusselt number vs. $\frac{DPe}{L}\left(=\frac{4}{\pi}N_{Gz}\right)$ for laminar or slug flow with sinusoidal wall heat flux distribution.

values and Equation (7), the entrance-region temperature profile was calculated and is shown in Figure 3. Since the diffusion equation and energy equation are analogous in form, these curves also represent the entrance-region concentration profiles for laminar mass transfer inside a round tube with constant mass flux at the wall.

Of some interest is the problem of finding the asymptotic expressions for β_n, $R_n(1)$, and C_n. From (6), the asymptotic solution of Equation (8), for large β and ξ close to unity is

$$R(\eta)=\frac{2\sqrt{2}\,\eta^{1/2}}{3}\left[\sin\left(\frac{\beta\pi}{4}-\frac{\pi}{3}\right)J_{1/3}\left(\frac{\sqrt{8}\,\beta\,\eta^{3/2}}{3}\right)-\sin\left(\frac{\beta\pi}{4}-\frac{2\pi}{3}\right)J_{-1/3}\left(\frac{\sqrt{8}\,\beta\,\eta^{3/2}}{3}\right)\right] \qquad (14)$$

where $\eta = 1 - \xi$. To satisfy the required condition, $\frac{dR}{d\eta} = 0$, at $\eta = 0$, the coefficient of the first term inside the bracket of Equation (14) must vanish. Accordingly

$$\beta_n = 4n + \frac{4}{3} \qquad (15)$$

which is the asymptotic expression for the eigenvalues for large n. By letting $\eta = 0$ ($\xi = 1$) in Equation (14) and making use of Equation (15), the asymptotic expression for $R_n(1)$ can be obtained as

$$R_n(1) = (-1)^n \frac{2^{1/3}}{3^{1/6}\Gamma(2/3)}\beta_n^{-1/3} = (-1)^n\, 0.774767\, \beta_n^{-1/3} \qquad (16)$$

Finally, by combining Equations (14), (13), and (15), the asymptotic expression for the C_n coefficient can be obtained as

$$C_n = (-1)^{n+1}\frac{3^{4/3}2^{4/3}\Gamma(4/3)}{\pi}\beta_n^{-4/3} = (-1)^{n+1}\,3.09955\,\beta_n^{-4/3} \qquad (17)$$

Equations (15), (16), and (17) can be used to predict the approximate values of β_n, $R_n(1)$, and C_n for large n. For n less than twenty, the error involved in using these approximation equations can be estimated by comparing the approximated values calculated from these equations with the accurate values listed in Table 1. For instance, the approximate eigenvalue for $n = 20$, calculated from Equation (15), is $\beta_{20}^2 = (81.3333)^2 = 6615.105689$, while the accurate value is (from Table 1), $\beta_{20}^2 = 6608.777727$. From Equations (16) and (17) the asymptotic values for $R_n(1)$ and C_n are 0.178765 and $-$ 0.0087958, respectively. These are in fair agreement with actual values of 0.18192 and $-$ 0.0089544.

DISCUSSION OF COMPUTATIONAL RESULTS

By using the obtained eigenvalues, the constants, Equation (3), and Equation (4), the local and average Nusselt numbers have been calculated for various values of $DPe/L\left(=\frac{4}{\pi}N_{Gz}\right)$, and are shown in Figures 4 and 5. Since preliminary calculations revealed the fact that even with the twenty accurately determined eigenvalues and the constants, the convergence of the infinite series appearing in the denominator of both Equations (3) and (4) were insufficient, it was decided to use the asymptotic eigenvalues for n larger than twenty. The asymptotic values are calculated by using Equation (15), and the corresponding eigenfunctions and the coefficients were evaluated based upon these eigenvalues by using available computer programs. Another possible approach would be to estimate the latter quantities by using Equations (16) and (17). Sixty-five eigenvalues were used to compute the Nusselt numbers, of which the last forty-five (n = 21 to 65) are those calculated from Equation (15). Inasmuch as the product $C_nR_n(1)$ becomes smaller for large n, the error involved in using the aymptotic values is believed to be of negligible magnitude. With the sixty-five eigenvalues and the constants, the infinite series showed satisfactory, though not complete, convergence. To demonstrate the magnitude of the error that might be encountered in stopping the infinite series at $n = 20$ rather than at $n = 65$, the local Nusselt numbers based upon these two types of calculation are shown and compared in Figure 6. It can be seen that the average deviation is $\approx$ 10%. If the infinite series were terminated at $n = 7$ (that is using the previously known eigenvalues only), the error could have reached as high as 30% for the local Nusselt numbers and $\approx$ 20% for the averaged Nusselt numbers. In Figure 7, the local Nusselt numbers for the case of slug flow are illustrated, based on the results calculated from Equation (5). For this case, the total num-

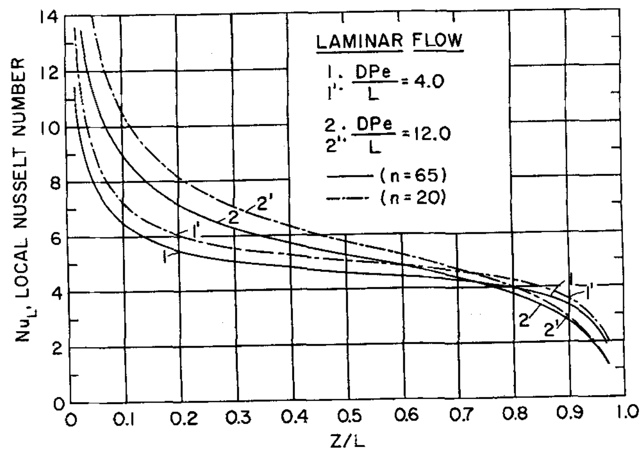

Fig. 6. Comparison of local Nusselt number for terminating the infinite series at n = 20 and n = 65.

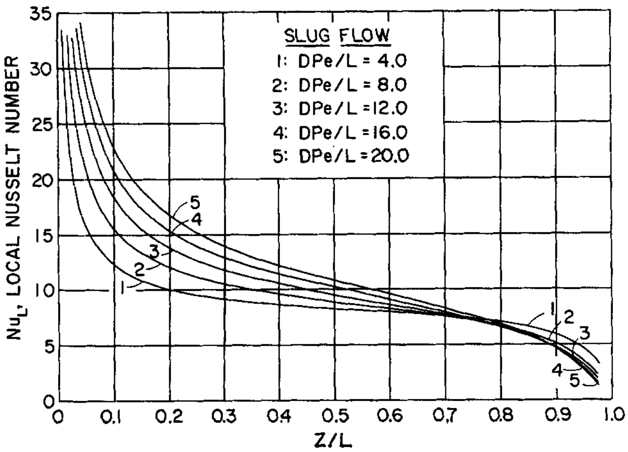

Fig. 7. Local Nusselt number vs. x/L for slug flow with sinusoidal wall heat flux distribution.

ber of the roots of Bessel function [that is the roots of $J_1(\alpha) = 0$] used in the computation was forty-five, with which the infinite series appearing in the denominator of Equations (5) and (6) showed adequate, though incomplete, convergence. Comparison of Figure 4 with Figure 7 reveals that, for sinusoidal heat flux distribution the local Nusselt numbers are generally much higher for the case of slug flow as compared to that for laminar flow. It will also be noticed that, in both cases the general behavior of the local Nusselt number is similar; all the curves are cross *s*-shaped. In Figure 5, the average Nusselt numbers for both laminar and slug flow are shown. The dotted line for laminar flow (curve 2′) shows the results based upon terminating the infinite series at $n = 20$, while the solid line (curve 2) represents the results based on sixty-five eigenvalues and the related constants. Once again, it can be noted that average Nusselt numbers for slug flow are generally $\approx$ 70 to 100% higher than those for laminar flow.

In Figure 8, the slug flow solution [Equation (5)] is compared with the turbulent flow experimental results of Petrovichev (5), who made heat transfer measurements for turbulent flow of mercury in a round duct with sinusoidal heat flux distribution along the length of the duct. By measuring the distribution of the longitudinal wall temperature, and calculating the bulk mercury temperature by making heat balances, he obtained some experimental local Nusselt numbers as shown in Figure 8. His measurements correspond to $Re = 10^5$, and $DPe/L \left(= \frac{4}{\pi} N_{Gz}\right) \cong 36$ and 55. The curves shown in Figure 8 are based on the slug flow solution [Equation (5)]. It

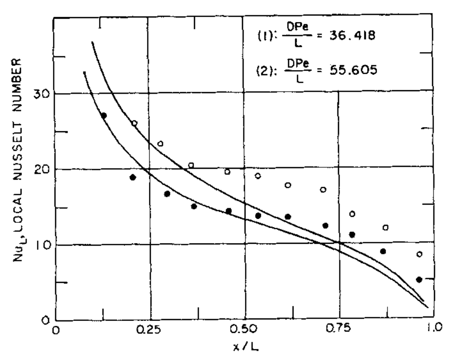

Fig. 8. Comparison of slug flow solution with turbulent flow experimental data of Petrovichev (5).

can be noted that for small values of x/L (for example the region not far from the duct inlet), the slug flow predictions agree rather closely with the turbulent flow results. Presumably, this is because in the duct entry region, the velocity profile has not fully developed and the slug flow condition is approximated. As x/L becomes larger, however, the experimental data tend to show much higher values as compared to the slug flow solution. The deviation is possibly caused by the gradual development of the velocity profile and the increased eddy transport of heat.

SUMMARY

In predicting the temperature field for heat transfer to laminar flow in a round tube with arbitrary longitudinal wall heat flux distribution, detailed information regarding the eigenvalues, eigenfunctions, and the coefficients of series expansion corresponding to the case of uniform wall heat flux are often necessary. Heat transfer with sinusoidal heat flux distribution constitutes one such example.

In this study, the first twenty eigenvalues and the associated constants were determined with sufficient accuracy with the aid of a high-speed digital computer. Furthermore, asymptotic expressions that may be used to augment these quantities for larger n's are obtained. This information was used to calculate both local and average Nusselt numbers for the case of sinusoidal heat flux distribution and were compared with the corresponding slug flow values.

ACKNOWLEDGMENT

The author wishes to thank Dr. O. E. Dwyer of Brookhaven National Laboratory for suggesting the problem and giving kind advice during the course of this study. This work was performed under the auspices of the U. S. Atomic Energy Commission.

NOTATION

A_n = $\beta_n^2/RePr$
C = heat capacity of fluid
C_n = coefficients of series expansion in Equation (7)
$J_{1/3}$ = Bessel function of first kind, 1/3 order
$J_{-1/3}$ = Bessel function of first kind, $-1/3$ order
J_1 = Bessel function of first kind, one order
L = length of a round tube
Nu_L = local Nusselt number as denfied by Equation (3)
$\overline{Nu}$ = average Nusselt number as defined by Equation (4)
N_{Gz} = Graetz number, $\left(= \frac{\pi}{4} \frac{DPe}{L}\right)$
Pr = Prandtl number, $(= C\mu/k)$
Re = Reynolds number, $(= 2r_o\overline{u}\rho/\mu)$
R_n = eigenfunction of Equation (8)
$R_n(1)$ = eigenfunction of Equation (8) at $\xi = 1$
T = temperature
T_o = fluid temperature for $x < 0$
T_b = bulk fluid temperature
T_{w_1} = wall temperature corresponding to uniform wall heat flux
a_n = $A_nL/\pi r_o = \beta_n^2/2N_{Gz}$
b_n = $\frac{4\alpha_n^2}{\pi}\left(\frac{L}{DPe}\right) = \alpha_n^2/N_{Gz}$
k = thermal conductivity of fluid
q_o = a constant wall heat flux
q_{total} = total heat flux through the tube wall
r = radial distance
r_o = radius of a tube

u = velocity of fluid
u_m = maximum fluid velocity in tube
$\overline{u}$ = average fluid velocity in tube
x = axial distance

Greek Letters

α_n = positive roots of $J_1(\alpha) = 0$
β_n = eigenvalues of Equation (8)
Γ = gamma function
η = $1 - \xi$
κ = $k/C\overline{u}\rho$
λ = parameter
μ = viscosity of fluid
ξ = normalized distance variable, $(= r/r_o)$

LITERATURE CITED

1. Brown, G. M., *A.I.Ch.E. J.*, **6**, No. 2, 179 (1960).
2. Carslaw, H. S., and J. C. Jaeger, "Conduction of Heat in Solids," Oxford Univ. Press, London, England (1958).
3. Dzung, L. S., Second United Nations Intern. Conf. Peaceful Uses of Atomic Energy, Geneva, **7**, 657 (1958).
4. Graetz, L., *Ann. Phys. Chem.*, **25**, 337 (1885).
5. Petrovichev, V. I., *Intern. J. Heat Mass Transfer*, **1**, 115 (1960).
6. Sellars, J. R., M. Tribus, and J. S. Klein, *Trans. Am. Soc. Mech. Engrs.*, **78**, 441 (1956).
7. Siegel, R., E. M. Sparrow, and T. M. Hallman, *Appl. Sci. Res.*, **A7**, 386 (1958).

Manuscript received November 25, 1964; revision received February 26, 1965; paper accepted March 1, 1965.

Optimum Design of Conventional and Complex Distillation Columns

J. M. SRYGLEY and C. D. HOLLAND

Texas A&M University, College Station, Texas

A method is proposed for achieving the optimum design, in the sense of minimum plates, for conventional and complex distillation columns for any set of specifications directly dependent on product purity which might be imposed by the designer. The method uses the calculational procedure of Thiele and Geddes, the θ-method of convergence, and sequential-search procedures. Illustrative examples chosen from a large number of design problems solved by this method are presented.

In the past, calculational methods that used successive approximations in the design of distillation columns were generally based on the calculational procedure of Lewis and Matheson (*10, 13*). These methods are essentially applicable only to the design of distillation columns for which the designer has set separation ratios of key components as the design criteria. Also, certain complex columns cannot be designed by these methods even for specifications of this type.

Hanson (*6*) and Holland (*7*) have proposed the indirect use of the calculational procedure of Thiele and Geddes (*15*) for the design of a column by solving problems for columns with various plate configurations until a solution is obtained or estimated that satisfies the design criteria. A new and more direct method is described herein for determining that plate configuration, for conventional or complex columns, which allows the specifications set by the designer, of any type directly dependent on product purity, to be met or exceeded with the minimum integral number of plates.

This new design method employs the procedure of Thiele and Geddes (*15*), the θ-method of convergence (*7, 11*), and multivariable sequential-search techniques (*1, 8, 16*). In this method, a functional expression or objective function is formed from the product specifications which, in a sense, measures the "distance" between the values specified for the products of the column and the values that are obtained for a particular plate configuration. Initially, sequential-search procedures are used to minimize this objective function for assumed temperature, liquid, and vapor-rate profiles in the column. The θ-method of convergence is then used to obtain new profiles in a manner which ensures that as many of the product specifications will be met exactly as can be obtained with an integral number of plates in each section of the column. The sequential-search procedure is continued until a minimum (relative or global) for the objective function is obtained. It should be noted that the proposed method, while not approximate, avoids the necessity of determining the minimum reflux ratio and the minimum total number of plates, quantities which are required by most approximate methods (*2, 3, 4*). For example, in the use of the proposed method in the design of conventional columns, two of the inputs consist of the ranges of the